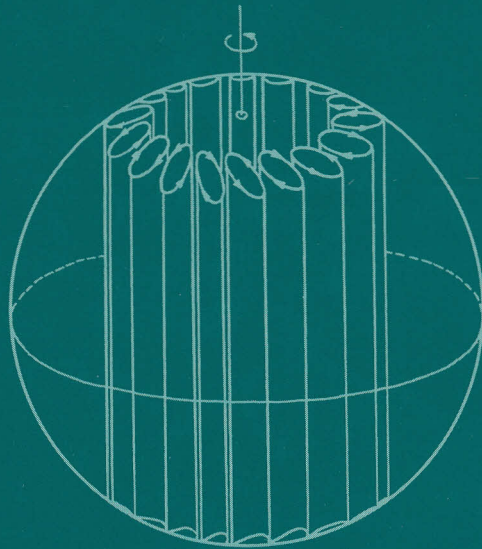


FLUID MECHANICS AND ITS APPLICATIONS

B.M. Boubnov and G.S. Golitsyn

# Convection in Rotating Fluids



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# Convection in Rotating Fluids

by

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